AD-A095 568 TEXAS UNIV AT AUSTIN CENTER FOR CYBERNETIC STUDIES F/G 5/9
MODEL EXTENSION AND COMPUTATION IN GOAL-ARC NETWORK APPROACHES --ETC(
DEC 80 A CHARNES, W W COOPER, A NELSON N00014-80-C-0242
UNCLASSIFIED CCS-391 NL

1 OF 1
AD
A095568

END
DATE
FILMED
3-81
DTIC

AD A095568

LEVEL

(9)

CENTER FOR CYBERNETIC STUDIES

The University of Texas
Austin, Texas 78712

Research Report CCS-391

MODEL EXTENSION AND COMPUTATION IN GOAL-ARC NETWORK APPROACHES FOR EEO PLANNING

by

A. Charnes
W.W. Cooper
A. Nelson
R. Niehaus*

December 1980

*Office of the Assistant Secretary of the Navy

This research was partly supported by ONR Contract N00014-80-C-0242 with the Center for Cybernetic Studies, The University of Texas at Austin. Reproduction in whole or in part is permitted for any purpose of the United States Government.

CENTER FOR CYBERNETIC STUDIES

A. Charnes, Director
Business-Economics Building, 203E
The University of Texas at Austin
Austin, TX 78712
(512) 471-1821

406197

ABSTRACT

Extensions of a "multi-modal" goal-arc EEO model are discussed in the context of prototype studies to check-out the model structure and computational efficiencies using PNET and other computer codes. The model presented provides for the information assistance needed after one has EEO goals to develop the strategies involving trade-offs between internal vs. external recruitment.

KEY WORDS

Equal Employment Opportunity

Manpower Planning

Flexible Transfers

Outside Recruitment

Mathematical Programming

Goal Programming

Markoff Process

Multi-Modal Networks

Network

Network Codes

Artifacts Goals

Goal-Arcs

A

INTRODUCTION

The model in this paper is an Equal Employment Opportunity (EEO) model for manpower planning. It has been developed in the context of manpower planning for the U. S. Navy's civilian work force. It is also applicable in other contexts such as industrial or construction work force planning, provided they have well-developed categories of employment. It is especially useful when promotion patterns need to be coordinated across a company--its divisions or its plants--across a period of time. The model and the methods of analysis are intended to be applicable in contexts where minority problems can take a variety of forms besides those encountered in the U. S.-- e.g., it should be useful for dealing with manpower planning in the context of the bi-cultural problems in Canada (see [12]).

Across a total company or a total government agency, such as the U. S. Navy, one may ignore fractional components of a solution. This is not true, however, at the plant level or at a construction site level where such fractional components cannot be ignored. Since the model is intended to deal with the problem of coordination within a total organization as well as the problem of manpower planning at a particular site, it becomes necessary to blend these two types of considerations together in a form that can be readily manipulated for interactive planning purposes in manpower planning as an additional aid to coordination of the components of a company, between company (or agency) wide and site level planning, or governmental agency. For concreteness, we relate these developments to the U. S.

Navy where they can be tied into a series of ongoing efforts that have already contributed important components to manpower planning research and practices.[1]

It is extremely important to relate these kinds of developments to the ultra high speed computer codes that are now available for network models. In this manner, we can do more than provide a means for integrating agency or company-wide manpower planning with decisions at the base (or site) level. We can also provide for interactive computation by means of which manpower planning decisions can be made a part of day-to-day decision-making activities. This can then open a new way for integrating decisions into the overall picture. The trick is to do this in a manner that facilitates coordination between on-the-site decision making and corresponding agency-wide planning without delay to the former.

In addition to serving overall management decision-making quality, it is also important to do this as part of Equal Opportunity planning since this is an area where long-range and short-range decision-making consequences need to be attended to in order to insure that overall EEO goals are being attained while attending to the other consequences.

Naturally, one expects some degree of conflict or inconvenience in many aspects of manpower planning. Hence, in common with earlier

---

[1]This can, for instance, be related to the flexible Equal Employment Opportunity models in [3] and [4].

efforts, we will continue to use a goal programming approach with embedded Markoff processes so that, on one hand, we can (a) take advantage of the accomplishments that are available from earlier research efforts, and (b) continue to relate these models to other parts of a continuing effort at the U. S. Navy.

The EEO models in some of these earlier efforts at the U. S. Navy required attention to the non-linearities which were handled by "goal artifacts" by means of which some of these non-linearities were moved into the functional. These non-linear components of the functional were then replaced by multiple arcs--which were called "goal arcs" because of their relation to the manpower goals that were being pursued. A sequence of such arcs could then be used to approximate the non-linear functional in a piecewise linear fashion (see [4] and [5]).

Proceeding in this manner offers additional advantages in that a network format can be achieved. As we shall see in the sections that follow, this network format will be developed analytically, after which we will introduce a "prototype problem" to facilitate understanding of what has been accomplished in these reductions to network form.

The term "prototype problem," as we use it, refers to a "realistic" problem in the sense that it contains the typical elements and correct orders-of-magnitude consequences for which the model is intended to be used. In this sense a "prototype" is realistic. It is

not to be regarded as a real problem, however, which would be of a size and complexity that would be likely to interfere with the succinct discussion and interpretations that are appropriate in an article of this kind. The further tests [1] that must precede actual implementation of our model in a real application also differ from the one we shall use here. Consistent with our use of a prototype example, we shall here concentrate on interpretation and illumination, but in a way that will also extend to the kinds of computational possibilities that are also a prior condition for implementation.

In this paper, we shall focus on a method of computation that involves a preemptive priority ordering of the race, nationality and sex (RNS) categories that need to be addressed in EEO planning. The procedure we employ is partly heuristic in character, but it does have the advantage of providing access to efficient solution procedures in ways that accord priority attention to minority needs--e.g., affirmitive action--over and above what might be sufficient in a simultaneous determination. Thus nothing is lost and something is gained because the route we are suggesting provides additional safeguards in an affirmative action context (by permitting assignments which give preemptive priority status to the minority aspect of manpower planning).

---

[1] Such tests are now being conducted at one of the Naval Air Research facilities.

The results achieved by our heuristic preemptive priority algorithm in our numerical example were also checked by exact comparison with a multicommodity network code developed by Jeff Kennington (see [10]). As we shall see, the results do not differ greatly in the present case. However, further research, which might be helpful in the general case of our preemptive priority algorithm, is indicated; although, as already noted, this algorithm should prove useful in EEO and similar planning contexts.

The plan of development for this paper may be summarized as follows: the bulk of the paper is concerned with the network development and prototype problem that will be examined in the immediately following sections. A concluding section will then try to summarize the supplemental issues and possible avenues of further research by reference both to the analytic development and the numerical example.

AN RNS GOAL-ARC MODEL

By an RNS Goal-Arc model we refer to a manpower planning model which takes Race, Nationality and Sexual distinctions into account via a network arrangement with Goal Arc features. Via the conventions introduced and developed in [5] we develop these network and goal arc features in the following manner.[1]

(1) To each job in each period we assign two nodes, an "antecedent" and a "consequent." We also designate as "job" nodes those corresponding to outside sources for recruitment (antecedents) and outside involuntary retirements (consequents). We also designate "job nodes" for normal organizational attrition (consequents). We designate the class of antecedent "job" nodes for period t as $J^-(t)$; the class of consequent "job" nodes by $J^+(t)$. $J_i^-(t)$ is the i^{th} job antecedent node; $J_j^+(t)$ is the j^{th} job consequent node.

(2) For each proper (real) job between two periods we designate a "valve" node to receive the goal arc flow from the consequent node of the immediate past period and to transmit an upper- and lower-bounded flow to the next period antecedent node. We let $V_i(t)$ denote the valve node for job i between periods t-1 and t.

(3) Finally, a super source node, S_0, and a supersink node, S_{n+1}, are added for CAPNET purposes.[2] The supersink node is connected back to the supersource node. Thereby, every node becomes a transshipment node.

---

[1] cf. Thompson [14] who also applies our Goal Arc approach to approximate the infinite programming of Grinold [9] by means of a finite network approach.

[2] See [8] for further discussions of this computer code and the considerable advantages it offers in computing solutions.

Before proceeding further, we pause to remark as follows: One of the analytical conveniences of the network approach is that it allows us to represent time flows in a very simple manner. To tie the dimension explicitly into our network representation we have introduced the terminology of antecedent and consequent nodes. In a similar vein we have introduced arcs in which flows are lower- and upper-bounded and refer to them as "valve arcs" with the function of ensuring that the flows remain within these limits. By this explicit structural representation we are then also able to assess the overall effects of these values by means of the dual evaluators associated with these lower and upper bounds.

The flow on every arc is unidirectional. Some of these may be "goal" arcs involving multiple arcs between the same two nodes for nonlinear functional approximation. They may also be single arcs. Also, every arc may have an upper and/or lower bound on its flows.

We now formalize these developments as follows: Let $x_{ij}^{\alpha k}(t)$ denote the flow from node $J_i^-(t)$ to $J_j^+(t)$ on the k^{th} individual arc of a multiple "goal arc" where each value of α refers to a pertinent RNS category. The corresponding lower and upper bounds are denoted by $L_{ij}^{\alpha k}(t)$ and $U_{ij}^{\alpha k}(t)$ respectively.

Let x_{0i}^{α} denote the flow from the supersource to the $J_i^-(1)$ for the α^{th} RNS category. Let $x_{i\ n+1}^{\alpha}$ denote the flow from $J_i^+(n)$ to the supersink for the α^{th} category. Let $x_{n+1\ 0}^{\alpha}$ denote the flow from the supersink to the supersource for the α^{th} RNS category.

Let $y_i^{\alpha r}(t)$ denote the flow on arc r of the goal-arc between $J_i^+(t - 1)$ and $V_i(t)$, $i \neq j_0$, for each α. The corresponding upper and lower bounds are $L_i^{\alpha r}(t)$ and $U_i^{\alpha r}(t)$. Let $\bar{y}_i(t)$ denote the flow on the "valve arc" between $V_i(t)$ and $J_i^-(t)$, $i \neq j_0$, for the α^{th} category

and let $y_{j0}(t)$ denote the flow between $J^+_{j_0}(t - 1)$ and $J^-_{j_0}(t)$.

The network node conditions, for all α, may now be written explicitly:

for supersource

(1) $x^{\alpha}_{n+1\ 0} - \sum_{i \varepsilon J^-(1)} x^{\alpha}_{0i} = 0$

for $i \ \varepsilon \ J^-(1)$

(2) $x^{\alpha}_{0i} - \sum_{j \varepsilon J^+(1)} \sum_k x^{\alpha k}_{ij} = 0$

for $j \ \varepsilon \ J^+(1)$

(3) $\sum_k \sum_{i \varepsilon J^-(1)} x^{\alpha k}_{ij}(1) - \sum_r y^{\alpha r}_j = 0, \quad j \neq j_0$.

For j_0 the "outside" node,

(4) $x^{\alpha}_{0j} + \sum_k \sum_{i \varepsilon J^-(1)} x^{\alpha k}_{ij}(1) - \sum_r y^{\alpha r}_{j0}(1) = 0$

Note that there is never flow from the "outside" node $J^-_{j_0}(t)$ to the natural attrition node $J^+_{i_0}(t)$.

For valve node $V_i(t)$

(5) $\sum_k y^{\alpha k}_i(t) - \bar{y}^{\alpha}_i(t) = 0,$

for $J^-_i(t)$, $t > 1$

(6) $\bar{y}_i^{\alpha}(t) - \sum_k \sum_j x_{ij}^{\alpha k}(t) = 0,$

for $J_j^+(t)$, $t > 1$

(7) $\sum_k \sum_{i \epsilon J^-(t)} x_{ij}^{\alpha k}(t) - \sum_r y_j^{\alpha r}(t) = 0,$

while for attrition node j_0,

(8) $\sum_k \sum_{i \epsilon J^-(t)} x_{ij_0}^{k}(t) - y_{j_0}(t) = 0,$

and finally, for the supersink, S_{n+1}

(9) $\sum_t y_{i_o}^{\alpha}(t) + \sum_{i \epsilon J^+(n)} x_{i,n+1}^{\alpha} - x_{n+1,0}^{\alpha} = 0.$

In addition to the above network conditions we also require that

$$\sum_{\alpha} \sum_k \bar{y}_i^{\alpha}(t) = b_i(t)$$

(10) $L_{ij}^{\alpha k}(t) \leq x_{ij}^{\alpha k}(t) \leq U_{ij}^{\alpha k}(t)$

$$L_i^{\alpha k}(t) \leq y_i^{\alpha k}(t) \leq U_i^{\alpha k}(t)$$

where $b_i(t)$ denotes the demand at $V_i(t)$ and the individual lower bounds, $L_{ij}^{\alpha k}(t)$ and $L_i^{\alpha k}(t)$, are all non-negative.

We now formulate our objective as follows:

(9) $\min \left[\sum_{\substack{i,j,k,t,\alpha \\ i \neq i_0}} c_{ij}^k x_{ij}^{\alpha k}(t) + \sum_{\substack{i,k,t,\alpha \\ i \neq i_0, j_0}} d_i^k y_i^{\alpha k}(t) \right]$,

and the minimization is subject to the conditions (1) through (10). Here the c_{ij}^{k}, $i \neq j_o$, are constants associated with the goal arcs for the approximating segments for the functional in going from antecedent to consequent nodes. When $i = j_o$, then $c_{joj}^{k} \equiv c_{j_oj}$ and $\sum_k x_{joj}^{\alpha k} \equiv x_{joj}^{\alpha}$ since there are no goal arcs in this case but only a penalty for hiring from the outside. When $j = j_o$, then $c_{0j_o}^{k} \equiv c_{0jo}$ and $\sum_k x_{ijo}^{\alpha k} \equiv x_{ijo}^{\alpha}$ since there are no goal arcs but only a penalty for firing. Finally the d_i^k are the slope constants for the linear segment of the functional associated with the $y_i^{\alpha k}(t)$. These result from the goal arc modelling conventions that we shall present in the next section.

GOAL ARC DEVELOPMENT

An illustration of the Goal-arc model is given in figure 1 for n time periods and m+2 job categories.[1] S_0 is the supersource node introduced on the left and S_{n+1} is the supersink node introduced on the right. In the diagram the antecedents and the consequents are represented by $J_{m+1}^{-}(t) = J_{j_o}^{-}(t)$, $J_{m+1}^{+}(t) = J_{j_o}^{+}(t)$ and $J_{m+2}^{+}(t) = J_{i_o}^{+}(t)$ respectively.

Some of the arcs represent natural flows and some may be goal arcs. Recall that the purpose of each of the latter, i.e., the goal arcs, is to represent a nonlinear goal functional element. To represent these piece-

---

[1]Adapted from [5].

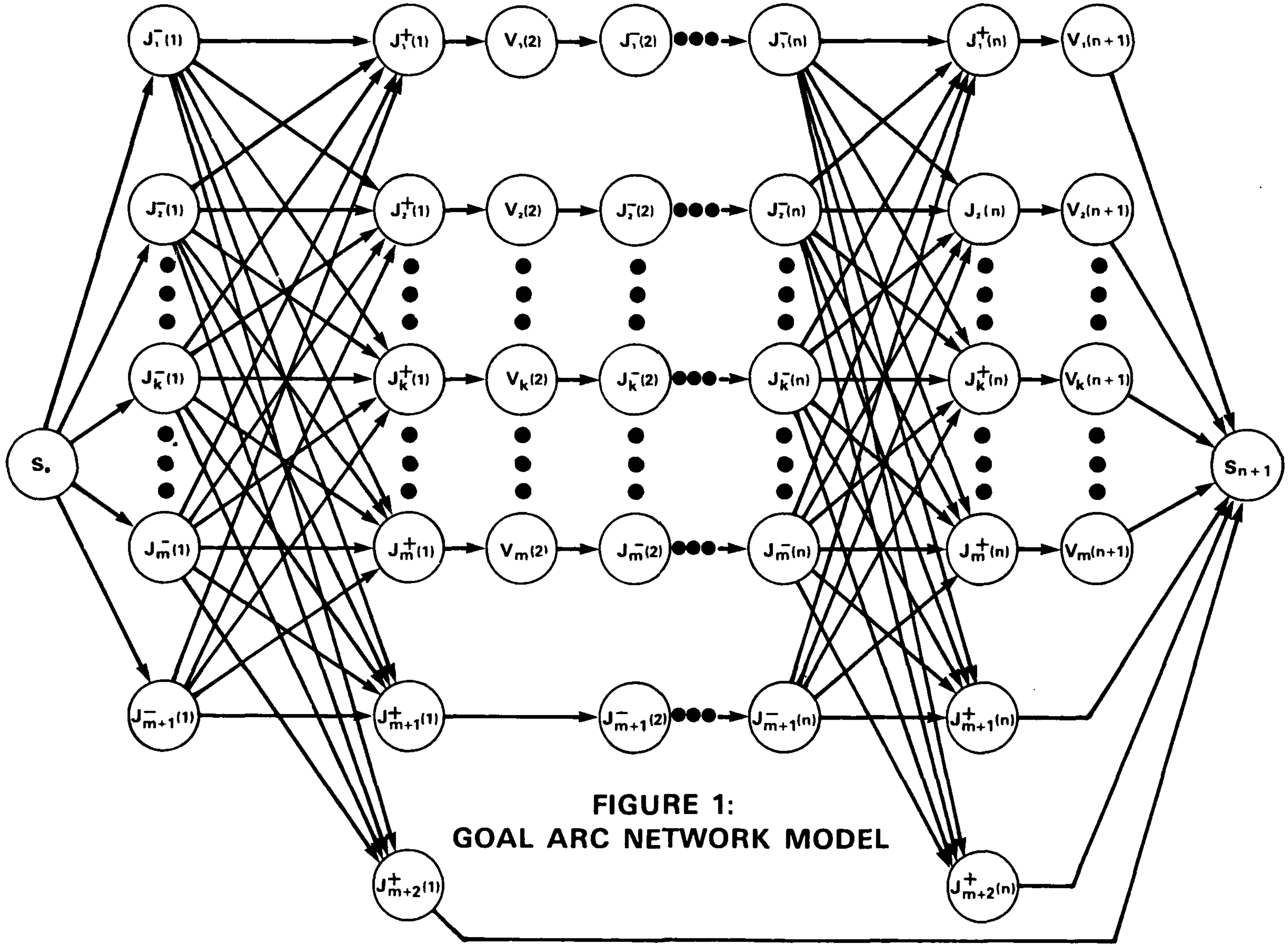

FIGURE 1:
GOAL ARC NETWORK MODEL

wise linear (nonlinear) goal elements we can replace each goal arc by multiple capacitated arcs between the same nodes.[1]

An illustration of a goal arc and its decomposition is in Figure 2.[2] Let the arc G between nodes N_1 and N_2 in the upper portion of Figure 2 be a goal arc. The decomposition of this arc is shown in the lower portion of Figure 2. The flow is broken up into flows z^k on G^k where $\sum_k z^k = z$. Each z^k is a bounded variable and z is the total flow. Further we let c^k be the slope assigned for the flow z^k. Thus, the decomposition of the piecewise linear representation of the nonlinear functional on the goal arc is accomplished. The single arc with nonlinear functional between N_1 and N_2 is replaced by a finite number of arcs with linear functionals on each.

PRECEDENCE ALGORITHM AND SOLUTION PROPERTIES

In various circumstances such as, e.g. affirmative action, one needs a procedure for introducing the various categories. When this is the applicable condition, then the appropriate solution for an RNS problem would proceed as follows. First one solves the problem for one α category in terms of the desired proportions as goals. Next, reducing the goals for the manpower program on the thus already assigned manpower, one proceeds to the next category. Proceeding in this order one continues until all the relevant categories have been assigned. For evident reasons we shall refer to this as the "precedence algorithm".

---

[1]For further detailed development of the underlying theory, see Charnes and Cooper [2], Chapter XVII.

[2]Adapted from [5]. See also Lewis [11].

DECOMPOSITION OF A GOAL ARC

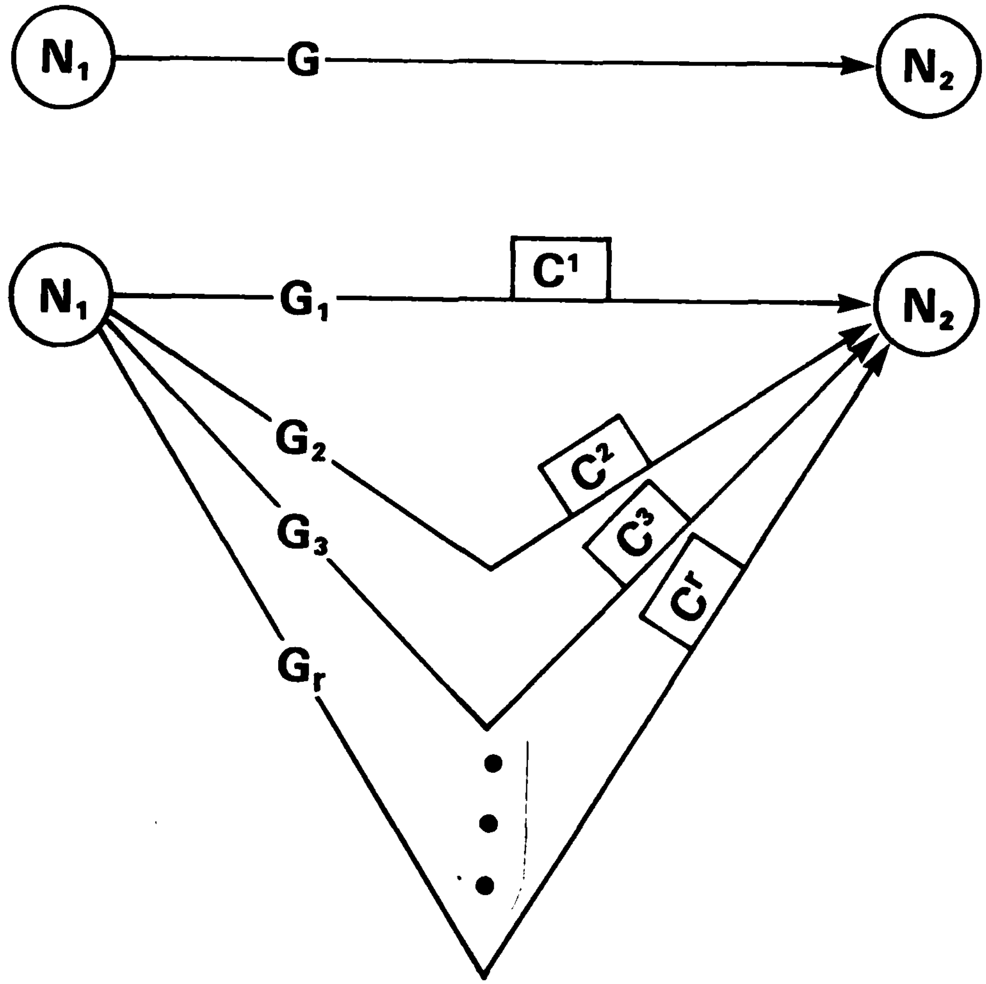

FIGURE 2

Such an algorithm is more efficient than the use of techniques that involve all RNS flows at every step of the calculation. If there is no specific order of priority one can apply the precedence algorithm several times in succession with different orders of precedence to uncover differences occasioned by these orderings. If the differences are small there is no need to use a complete multi-modal algorithm.

Note that as soon as one decomposes across-RNS capacities into individual modal capacities then the complete RNS problem also decomposes into single separate problems. As long as these individual capacities are the same, one gets the same results no matter what order of precedence is used. Thus, one might anticipate that in many cases the precedence order will be immaterial for a precise solution to the total RNS problem.

In the example we shall discuss it was found that by using different orders of precedence only very small changes resulted.[1]

The results of the precedence algorithm were also compared with the multi-modal algorithm of Kennington and Shalaby [10] as adapted to the RNS problem in this example. Again there were no substantial differences. Hence by virtue of this cross check--as well as by the other reasons that have already been given--we can be reasonably sure that the precedence algorithm is giving desired results in our example.

One of the most important aspects of the goal arc methodology is its association with the integer-solution property. This is important because of the small cell sizes that one sometimes encounters.[2] Recall that projection of expected transfers between positions in the organization

---

[1]Considerably less than 1%.

[2]See [4] and [5].

are described by Markoff transition matrices with elements that are generally not integers. Thus, their projections are also not integers generally. For integer results in our goal arc model, however, we must have these projections in integer terms.

Thus a problem arises as to how to secure projections that are integers. Two possibilities are "rounding up" and "rounding off".

With "rounding up" one provides more opportunities for transitions. On the other hand without further restrictions these can produce numbers of persons to be transferred that deviate widely from transfers that are normally possible.

To handle this situation with "rounding up" one may impose individual upper bounds on transfer flows such that the latter will be of reasonable size. Other methods such as "rounding off" will lead to more conservative results--i.e. fewer transfer opportunities would result from "rounding off" than "rounding up".

Examples of these and other possibilities will be supplied along with other interpretations and comments in the context of the numerical (prototype) illustration that follows.

NUMERICAL ILLUSTRATION

In order to illustrate the extended Goal-Arc model, we will now consider an example synthesized from numerical data as follows.

Let there be four RNS categories $\alpha = 1,2,3,4$ (minority female, white female, minority male, white male) and five time periods. For job categories we use the following:

Subscript (i,j,)	Abbreviation (Clerical, Technical, Administrative)	Description (Category and Level)
1	C1	Clerical-level 1
2	C2	Clerical-level 2
3	T1	Technical-level 1
4	T2	Technical-level 2
5	T3	Technical-level 3
6	A2	Administrative-level 1
7	A3	Administrative-level 2

We also represent outside source and natural attrition by "O" and "NA" respectively.

Figure 3 provides the initial "onboard" and "targeted" work-force goals, $b_i(t)$, where i = 1,....,7 for the associated job category for the first and fifth time periods. Thus, e.g., the goals for clerical level one in periods 1 and 2 are respectively 397 and 393 where the start occurs with 411 persons on board in period zero. I.e. we take the number on board as our period 0 goal. "Period 1" and "Period 2" represent generic characterizations. Thus "period 1" refers to projected goals for one year while "period 2" is a 5-year projection to illustrate jointly "short term" and "long term" uses of the model without undue distraction of numerical detail.

	0	1	2
C1	411	397	393
C2	210	203	201
T1	54	52	52
T2	235	227	224
T3	532	514	508
A2	183	177	174
A3	69	66	62

WORKFORCE GOALS

Figure 3

With these same conventions Figure 4 provides a further breakdown of these goals by RNS categories. Again the period 0 goal represents the number onboard so that e.g. summing over C1 for minority female (=83), white female (=279), minority male (=18), and white male (=31), we obtain the 411 persons listed in the upper left hand corner of Figure 3. The EEO goals are obtained from the Navy's EEO goals determination system.[1] Linear interpolation is used to determine the goals for the other time periods.

	WHITE MALE (α=1)			WHITE FEMALE (α=2)		
i \ t	0	1	2	0	1	2
C1	31	29	28	279	275	297
C2	7	8	11	167	162	160
T1	22	23	30	18	16	14
T2	128	124	127	66	62	56
T3	440	418	361	17	17	23
A2	138	133	132	16	15	12
A3	36	35	28	32	30	29

	MINORITY MALE(α=3)			MINORITY FEMALE (α=4)		
i \ t	0	1	2	0	1	2
C1	18	18	16	83	75	52
C2	5	4	7	31	29	23
T1	7	6	3	7	7	5
T2	24	25	27	17	16	14
T3	74	75	98	1	4	26
A2	21	21	21	8	8	9
A3	0	0	3	1	1	2

Figure 4

[1] See [1]. The data used in this example were obtained from one of the early experimental systems used to help in forming this system.

M, the markoff matrix of transition probabilities that we shall use is presented in Figure 5 where the shading indicates places where transitions do not occur. The coefficients in Figure 5 are carried out to 3 decimal places to illustrate some of the small cell size problems we have already referred to. For instance the .001 probability of going from A2 to A3 is a case in point which we shall examine along with other aspects of the numerical results obtained from Figure 5.

TRANSITION RATES

To \ From	C1	C2	T1	T2	T3	A2	A3
C1	.699	.020	.050	.003		.009	
C2	.091	.788	.006	.008		.030	
T1	.013	.001	.371				
T2	.008	.005	.268	.723	.004	.038	
T3			.002	.124	.874	.009	.014
A2	.004	.030	.006	.020	.001	.511	.001
A3		.002		.017	.044	.315	.872
Attrition	.190	.154	.297	.105	.077	.088	.113

FIGURE 5

The "artifact goals", it may be recalled,[1] are defined by

(10) $$g^{\alpha}_{ij}(t) = \langle b^{\alpha}_{i}(t-1)M_{ij} \rangle ,$$

where M_{ij} is the i,jth element of M and $\langle u \rangle$ represents the smallest integer not less than u. The "artifact goals" for each RNS category are then calculated as indicated by (10).

[1] See [5].

REDUCTION TO RNS NETWORK FORMAT

We now formulate this problem as an RNS network. This is done by describing the decomposition of the goal arcs and the setting of upper and lower bounds on the other arcs.

There are two types of "goal arcs" represented respectively by arcs between nodes $J_i^+(t)$ and $J_j^-(t)$, and the arcs between $J_j^-(t)$ and $V_j(t)$, where i and j represent real job categories. In this example we will employ only two pieces in our piecewise linear goal functional, i.e. k = 2 in the decomposition of the "goal arcs" in order to present the decomposition of the "goal arcs" with examples.

Consider the "goal arcs" between $J_i^+(t)$ and $J_j^-(t)$. Let i=1, j=2, t=1 and α=1. The arc $G_{12}(1)$ between $J_1^-(1)$ and $J_2^+(1)$ is a goal arc. It is replaced by two arcs, say $G_{12}^1(1)$ and $G_{12}^2(1)$. Now $x_{12}^{1k}(t)$ denotes the number of transfers for minority female from clerical level 1 to clerical level 2 via the $k^{\underline{th}}$ route in time period 1. These transfers are bounded as follows:

$$0 \leq x_{12}^{11}(1) \leq 2$$ [1]

$$0 \leq x_{12}^{12}(1) < \infty .$$

Let c^k denote the functional coefficient on G_{ij}^k, where k = 1,2. We assume that $c^1 < c^2$. Hence we have the decomposition of the "goal arc" between $J_1^-(1)$ and J_2^+ for minority female. All other "goal arcs" between $J_i^+(t)$ and $J_j^-(t)$ for all RNS categories are decomposed in a similar manner.

Now consider the "goal arc" $G_1(1)$ between $J_1^+(1)$ and $V_1(1)$. As above

---

[1] $g_{12}^1(1) = \langle .02 \times 83 \rangle = \langle 1.66 \rangle = 2$

we replace this arc with two arcs, $G_1^1(1)$ and $G_1^2(1)$. Now let $y_1^{1k}(1)$ denote the number of minority females in the clerical level 1 job category at the end of period 1 who got there via the $k^{\underline{th}}$ route. The $y_1^{1k}(1)$ are bounded as follows:

$$0 \leq y_1^{11}(1) \leq 75$$ [1]

$$0 \leq y_1^{12}(1) < \infty.$$

Let d^k denote the functional coefficient for the flow on $G_i^k(t)$, with $k = 1,2$. We assume that $d^1 < d^2$. Hence we have decomposed the "goal arc" between $J_1^+(1)$ and $V_1(1)$ for minority females in the first time period.

We describe how the upper and lower bounds for flow on the non-goal arcs are set. The bounds on arcs between $J_i^+(t)$ and the node for natural attrition, NA, for the $\alpha^{\underline{th}}$ RNS category are set at $g_{ii_0}^{\alpha}(t)$, for example for the arc between $J_1^+(1)$ and NA for minority females the upper and lower bounds are set at 16.[2] For arcs from $J_i^+(t)$ to the "outside" node and from "outside" node to $J_i^-(t)$ the lower bounds are set at zero and the upper bound at ∞ for each RNS category. Finally, the lower and upper bounds for the valve arcs, i.e. arcs from $V_i(t)$ to $J_i^+(t+1)$ and from $V_i(5)$ to the supersink for each ethnosexual category are set, respectively, at the projected manpower requirement for that ethnosexual category plus ten percent and minus ten percent of the requirement. For example the lower and upper bounds on the valve arc between $V_1(1)$ and $J_1^+(2)$ for minority females are 67 and 83, respectively.

There are mutual capacity constraints on the arcs between $V_i(t)$ and

---

[1] 75 is the EEO goal for minority female in the clerical-level 1 job category in the first time period.

[2] $g_{11_0}^1(1) = \langle 83 \times (.19) \rangle = \langle 15.8 \rangle = 16.$

$J_i^+(t+1)$ and between $V_i(5)$ and the supersink. For the clerical-level 1 job category in the first time period the targeted work force goal is 397. Note that $\bar{y}_1^\alpha(1)$ denotes the EEO goal for the clerical-level 1 job category in the first time period for the $\alpha^{\underline{th}}$ RNS type. The mutual capacity constraint in this case is

$$\sum_\alpha \bar{y}_1^\alpha(1) = 397.$$

Proceeding in this manner the problem is represented as a network problem with the "goal arcs" decomposed.

Since the objective function is to be minimized, the larger the value of the functional coefficient on an arc the greater the resistance to flow on that arc. In our penalty system, the following priorities are established:

1. Meeting the goal of a certain number of personnel of each RNS type in each job category in each time period is given the highest priority.
2. The expected movement between job categories has the second highest priority.
3. The penalty on exceeding manpower requirements for each RNS category is greater than any other penalty except the penalty for firing.
4. Firing is highly discouraged.
5. The penalty on hiring is less than the penalty on exceeding manpower requirements but greater than the penalty on expected movements.
6. The penalty on firing is set at an order of magnitude larger

than the sum of all other penalties.

The value for the functional coefficients on the arcs are given as follows

SYMBOL	DESCRIPTION	PENALTY
P	Penalty on flexible movement	2
R	Firing penalty	1000
H	Hiring penalty	5
G	Penalty on expected movement	-1
Q	Penalty on meeting manpower requirements	-6
F	Penalty on exceeding manpower requirements	10

Results from the above data and modeling procedures, in which "rounding up" was used to determine the "artifact goals", were attained with the precedence algorithm. As noted earlier these results were obtained under various orders of precedence and further confirmed by means of the multi-modal algorithm of Kennington and Shalaby (see [10]). We regard as optimal the results obtained with the precedence algorithm.

SOLUTION RESULTS

To facilitate discussion, the results of only one of the four RNS groups are presented here. The results are shown for minority males, over three time periods, the first, third and fifth. Table 1 provides a summary of the personnel actions projected by the precedence algorithm which will be used to guide the discussion. The results contained in this table are also interpreted. Finally a detailed analysis of the internal transfers are examined with special attention being given to the notion of flexibility options and bridge positions.

As noted above, Table 1 summarizes the projected personnel actions for minoirty males in the first, third and fifth time periods. In this table the following information is furnished: (1) the initial onboards, (2) the internal and external gains and losses, (3) the final onboards, (4) the goals and (5) the deviations from the goals. The internal gains and losses indicate, respectively, transfers into and transfers out of job categories. For further explication we may observe that external gains and losses indicate, respectively, hires and attritions for job categories. Attritions, we should note, however, can be of two types: involuntary and voluntary.

Involuntary attritions are simple fires. Voluntary attritions include such things as retirements, individuals taking jobs outside the organization or leaving of their own volition for other reasons, etc.

TABLE 1

SUMMARY OF PERSONNEL ACTION
MINORITY MALE

	OCCUPATION	INITIAL IN-BOARD	GAINS INTERNAL	GAINS EXTERNAL	LOSSES INTERNAL	LOSSES EXTERNAL	FINAL ON-BOARD	GOAL	DEVIATION
PERIOD 1									
	C1	18	1	6	4	3	18	18	0
	C2	5	2	0	2	1	4	4	0
	T1	7	1	3	3	2	6	6	0
	T2	24	7	0	3	3	25	25	0
	T3	74	3	7	3	6	75	75	0
	A2	21	5	1	4	2	21	21	0
	A3	0	0	0	0	0	0	0	0
TOTAL		149	19	17	19	17	149	149	0
PERIOD 3									
	C1	18	1	5	3	3	18	18	0
	C2	4	2	0	1	1	4	4	0
	T1	6	1	5	4	2	6	6	0
	T2	25	6	0	3	3	25	25	0
	T3	79	3	10	3	6	83	83	0
	A2	21	6	1	5	2	21	21	0
	A3	0	0	0	0	0	0	1	-1
TOTAL		153	19	21	19	17	157	158	-1
PERIOD 5									
	C1	18	3	0	2	3	16	16	0
	C2	4	1	5	2	1	7	7	0
	T1	6	1	0	2	2	3	3	0
	T2	25	8	0	3	3	27	27	0
	T3	87	3	18	3	7	98	98	0
	A2	21	1	9	8	2	21	21	0
	A3	0	3	0	0	0	3	3	0
TOTAL		161	20	32	20	18	175	175	0

In these terms Table 1 represents actions that can be taken by management and the projected consequences of those actions. For example, for minority males in the first time period, the following actions are suggested by our solution for the clerical level-1 job category: (1) hire 6 (= External Gains), (2) transfer 1 into this job category (= Internal Gain), (3) transfer 4 out of this job category into other jobs (= Internal Loss) and finally (4) allow for external losses of 3 persons from voluntary and involuntary attritions. Initially there were 18 persons in the clerical level-1 in the first time period; and, as a consequence of the projected actions, there will be 18 on hand at the end of this time period. In this case there is no deviation from the goal which was 18.

Obviously, the results for all job categories and time periods can be interpreted in a similar manner. Although not reported in Table 1, the results of the precedence algorithm provide detailed information on the internal movements and transfers in the organization. Furthermore, a distinction can be (and is) made in the model between normal transfers, i.e., transfers expected from historical experience, and flexible transfers, i.e., transfers different from historical experience. The latter are referred to as "flexible transfers" because they allow for the use of additional opportunities that can be accomodated on so-called "bridge positions."[1] Via the erection of such bridge

---

[1]See Kathy Lewis [11] for additional discussion that relates this to the concept of "organizational slack." It represents not static amounts of slack as in Cyert and March [7] , however, but is rather the slack that develops dynamically as a flow unfolds. Thus the model provides a way in which such dynamically developing slack can be monitored by top (or central) management and integrated into its overall EEO plan instead of just being left to lower level management bargaining as in Cyert and March [7] .

positions we are thereby enabled to exploit what might be termed "targets of opportunity" which might otherwise be lost to EEO plans--and to do so without violating legal conditions or civil service regulations.

We can explain what is involved by reference to Table 2, which supplies supporting detail on the changes summarized in Table 1. Consider, for instance, the row labelled A2 in Table 2. The column headings indicate the expected and projected values of transfers from A2 to the categories indicated by the titles of these columns in Table 2.

Turning to the column labelled "Exp" under A2 we have a total of 11 persons who are expected to remain in this category to which another 5 should be added as transfers in from other categories. Thus, a total of 16 persons are expected to be in this category as a result of internal conditions only.

This represents the situation that would have been obtained for A2 in Period 3 as a result of internal conditions only. We now relate this to Table 1 to which these data are pertinent for the A2 category in Period 3. In Table 1 we can observe that an external gain of one person (i.e. a new hire is also planned) which would result in an expected total of 16 + 1 = 17 persons at the A2 level at the end of period 3.

The Proj value under A2 includes the flexibility option choices from which we see that the transfers from T3 to A2 are augmented from 1 to 2 persons and, similarly, the number who stay in A2 are augmented from 11 to 14 persons. Hence, the expected internal values give way to the projected value of 20. No change in external gains is projected beyond the already expected increase of 1 person which added to 20 gives the value of 21 persons exhibited as the Final On-Board value for the A2 category in period 3.

TABLE 2

PROJECTED PERSONNEL TRANSFERS
MINORITY MALES

PERIOD 3

TO / FROM	C1		C2		T1		T2		T3		A2		A3	
	EXP	PROJ	EXP	PROJ	EXP	PROJ	EXP	PROJ	EXP	PROJ	EXP	PROJ	EXP	PROJ
C1	13	12	0	0	1	1	1	1	0	0	1	1	0	0
C2	1	0	4	2	1	0	1	0	0	0	1	1	1	0
T1	1	0	1	1	3	3	2	2	1	0	1	1	0	0
T2	1	0	1	0	0	0	19	19	4	2	1	1	1	0
T3	0	0	0	0	0	0	1	1	70	70	1	2	4	0
A2	1	1	1	1	0	0	1	2	1	1	11	14	7	0
A3	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Thus, in this one category we find that the number of minority males is augmented from 17 as an expected number to 21 as a projected value in period 3. The transfer of an added person from T3 to A2 is accomplished via a "bridge position," at least in principle.[2] The projected retention from 11 to 14 persons in A2 does not require a bridge position. However, both augmentations result from the use of the flexibility option that the model provides.

In a similar manner, the flexibility option extends to the augmented losses from each category. Note, for example, that the expected loss from A2 to T2 in Table 2 is one person but the flexibility option projects this to an autmentation of 2 persons.

Evidently, the exercise of the flexibility options must simultaneously consider all such gain and loss possibilities in the way these augementations are used in moving toward the manpower planning and EEO goals. This involves complex, combinatorial considerations that are likely to require recourse to computerized mathematical models for their full exploitation. Even the issue of round off can have effects which also require explicit treatment.

Evidently, the exercise of the flexibility options must simultaneously consider all such gain and loss possibilities in the way that these augmentations are to be used in moving toward manpower planning and EEO goals. This involves complex, combinatorial considerations that are likely to require recourse to computerized mathematical models for their full exploitation. Even the issue of round off can have effects which also require explicit treatment.

---

[2]See [11, pp. 40 ff.] for further discussion.

TABLE 3

MINORITY MALES

	OCCUPATION	INITIAL ON BOARD	GAINS INTERNAL	GAINS EXTERNAL	LOSSES INTERNAL	LOSSES EXTERNAL	FINAL ON BOARD	GOAL	DEVIATION
PERIOD 1									
	C1	18	0	3	0	3	18	18	0
	C2	5	0	0	0	1	4	4	0
	T1	7	0	3	2	2	6	6	0
	T2	24	8	0	4	3	25	25	0
	T3	74	4	3	0	6	75	75	0
	A2	21	0	8	6	2	21	21	0
	A3	0	0	0	0	0	0	0	0
TOTAL		149	12	17	12	17	149	149	0
PERIOD 3									
	C1	18	0	4	1	3	18	18	0
	C2	4	1	0	0	1	4	4	0
	T1	6	0	4	2	2	6	6	0
	T2	25	3	4	4	3	25	25	0
	T3	79	3	7	0	6	83	83	0
	A2	21	1	2	1	2	21	21	0
	A3	0	0	0	0	0	0	1	-1
TOTAL		153	8	21	8	17	157	158	-1
PERIOD 5									
	C1	18	0	3	2	3	16	16	0
	C2	4	3	1	0	1	7	7	0
	T1	6	0	1	2	2	3	3	0
	T2	25	9	0	4	3	27	27	0
	T3	87	3	18	3	7	98	98	0
	A2	21	1	9	8	2	21	21	0
	A3	0	3	0	0	0	3	3	0
TOTAL		161	19	32	19	18	175	175	0

Table 3 will help us to illustrate what is involved. This table is the one that would result if the usual round-off rules were used. It is to be contrasted with the data of Table 1 which is the plan that emerges when "round-up" is used instead of "round-off." Note, for instance that the Period 3 value for internal gains in A2 is only 1 person in Table 3, but in Table 1 this is expanded to a projected value of 6 persons. Evidently, the latter provides a great deal more mobility than the former and the same situation attains throughout the Tables 1 and 3 comparisons.

How far and to what extent these "round-up" and "round-off" differences can (or should) be used as guides to manpower planning evidently also requires mathematical controls for proper evaluation. They cannot be ignored, however, without missing possibly increased mobility opportunities. Furthermore, the model should be capable of handling additional constraints of a policy or budgetary variety, and it should be capable of doing this in an easily implementable manner.

CONCLUSION

Evidently, the model presented in this paper has the kinds of capabilities we have just illustrated. It has been formulated in network form via the Goal-Arc formalisms that were introduced in [5]. This provides access to the highly efficient network codes such as PNET [8] or D. Klingman's CAPNET for computation and implementation of the very large models that are likely to be involved in dealing with all of the RNS categories in an integrated manpower-EEO planning system.

An algorithm called the "precedence algorithm" was also presented that provides access to the CAPNET code. The results in efficiency increases of at least an order of magnitude relative to direct employment of available multi-modal codes. Additionally, it provides information not available from the multi-modal algorithms, since it reflects the results of prioritizing the EEO categories relative to one another.

Checks thus far with multi-modal algorithms such as the one provided by Kennington and Shalaby in [10] have resulted in maximum differences of less than 1% in the assignments. Addition of other constraints such as those of a budgetary variety may alter this situation. This is a subject for further research. In the meantime, the models and methods we have provided can be used not only for manpower planning procedures involving EEO and affirmative action considerations in both government and private enterprise contexts in the U.S. but also for related problems such as manpower planning in contexts like the bicultural situations of Canada.

REFERENCES

[1] Atwater, D.M., R.J. Niehaus, and J.A. Sheridan, "External Labor Market Analysis and EEO Goal Planning", OCP Research Report No. 33, (Washington, D.C.: U.S. Navy Office of Civilian Personnel, 1978).

[2] Charnes, A. and W.W. Cooper, Management Models and Industrial Applications of Linear Programming, (New York: John Wiley and Sons, Inc., 1961).

[3] ________, ________, K.A. Lewis, R.J. Niehaus, "A Multi-Objective Model for Planning Equal Employment Opportunities" in M. Zeleny, ed., Multiple Criteria Decision Making: Kyoto, 1975, (New York: Springer Verlag, 1976).

[4] ________, ________, ________, ________, "A Multi-Level Coherence Model for Equal Employment Opportunity Planning", in A. Charnes, W.W. Cooper and R.J. Niehaus, ed., Management Science Approaches to Manpower Planning and Organization Design, (New York: Elsevier-North Holland, 1978).

[5] ________, ________, ________, A. Nelson and R.J. Niehaus, "A Nonlinear Goal-Arc Network Extension of the Multi-Level Coherence Model for EEO", CCS Research Report No. 296 (Austin, TX: Center for Cybernetic Studies, University of Texas, November 1977).

[6] ________, ________ and R.J. Niehaus, Studies in Manpower Planning, (Washington, D.C.: U.S. Navy Office of Civilian Manpower Management, 1972).

[7] Cyert, R.M., J.G. March, "Organizational Factors in the Theory of Oligopoly," Quarterly Journal of Economics, Vol. 70, 1956.

[8] Glover, G., Karney, D. and D. Klingman, "Implementation and Computational Comparison of Primal, Dual and Primal-Dual Computer Codes for Minimum Cost Network Flow Problem", CCS Research Report No. 136 (Austin, TX: Center for Cybernetic Studies, University of Texas, July 1973).

[9] Grinold, R.C., "Interactive Design of a Manpower System", in T.I. Bryant and R.J. Niehaus, ed., Management Science Approaches to Manpower Planning and Organization, (New York: Elsevier-North Holland, 1978).

[10] Kennington, J. and M. Shalaby, "An Effective Subgradient Procedure for Minimal Cost Multicommodity Flow Problems", Management Science, Vol. 23, No. 9, 1977, pp. 994-1003.

[11] Lewis, K.A., Manpower Planning for Equal Employment Opportunities as Applied to the U.S. Navy Civilian Workforce, unpublished Ph.D. dissertation (Pittsburgh, PA: School and Urban and Public Affairs, Carnegie-Mellon University, 1977).

[12] Mayhood, J.E., C.A.U. Cotton and W.L. Price, "Planning Manpower Utilization in a Bi-Cultural Environment" in D. Bryant and R.J. Niehaus, ed., Management Science Approaches to Manpower Planning and Organizational Design, (Elsevier-North Holland, 1978).

[13] Niehaus, R.J., Computer-Assisted Human Resources Planning, (New York: Wiley-Interscience, 1979).

[14] Thompson, G.L., "Network Transshipment Model for Manpower Planning and Design" in Y. Ijiri and A.B. Whinston, ed., Quantitative Planning and Control, (New York: Academic Press, 1979).

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: CCS 391	2. GOVT ACCESSION NO.: AD-A095 568
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): Model Extension and Computation in Goal-Arc Network Approaches for EEO Planning.	5. TYPE OF REPORT & PERIOD COVERED
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): A. Charnes, W.W. Cooper, A. Nelson, and R. Niehaus	8. CONTRACT OR GRANT NUMBER(s): N0014-80-C-0242
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Center for Cybernetic Studies, UT Austin, Austin, Texas 78712	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: Office of Naval Research (Code 434), Washington, D.C.	12. REPORT DATE: December 1980
	13. NUMBER OF PAGES: 35
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

This document has been approved for public release and sale; its distribution is unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Equal Employment Opportunity, manpower planning, flexible transfers, outside recruitment, mathematical programming, goal programming, markoff process, multi-modal networks, network, network codes, artifacts goals, goal-arcs

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Extensions of the "multi-modal" goal-arc EEO model are discussed in the context of prototype studies to check out the model structure and computational efficiencies using PNET and other computer codes. The model presented provides for the information assistance needed after one has EEO goals to develop the strategies involving trade-offs between internal vs. external recruitment.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

ATE
LMED
8